AD-A099 252 NAVAL COASTAL SYSTEMS CENTER PANAMA CITY FL F/G 17/1
A STATISTICAL APPROACH TO PASSIVE TARGET TRACKING.(U)
APR 81 M J HINICH
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(14) NCSC- REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TM 311-81	2. GOVT ACCESSION NO. AD-A099252	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) A Statistical Approach to Passive Target Tracking.		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Melvin J. Hinich		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Coastal Systems Center Panama City, FL 32407		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS (17) Task Area SF 34 371 691 Task 16455-2
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Coastal Systems Center Panama City, FL 32407		12. REPORT DATE (11) Apr[redacted]981
		13. NUMBER OF PAGES 21
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) 26		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

(16) F34371

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Coordinates
Tracking
Simulation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A method for passively tracking a moving target using a sequence of bearings from a surveillance platform is presented. The key assumption of the method is that the target is moving at a constant speed on a fixed heading during the data acquisition period. The procedure can be considered as a generalization of Ekelund ranging since parameter estimates are computed after bearings are taken as the tracking platforms maneuvers. No constraint is placed on the track of the platform. The estimators presented in this paper are approximately maximum likelihood when the target is distant from the platform.

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INTRODUCTION

This report presents a method for estimating the coordinates of a moving target as a function of bearing direction cosines measured from a tracking platform. The procedure can be considered as a generalization of Ekelund ranging since parameter estimates are computed after bearing measurements are taken during a period when the tracker maneuvers. The key assumption for the method is that the target is moving at a constant speed on a fixed heading. This is the same assumption made for Ekelund ranging. In contrast, however, the method which is the subject of this report does not constrain the course of the tracking platform.

DEVELOPMENT OF THE TRACKING PARAMETERS

For a fixed coordinate system, let $x_T(t)$ and $x_B(t)$ denote the x coordinates of the target and tracker at time t. Let $y_T(t)$ and $y_B(t)$ denote their y coordinates. Let B(t) denote the true target bearing at time t measured with respect to the y axis. For example, a target on the y axis would be at 0 or 180 degrees. Then, from Figure 1,

$$\sin B(t) = \frac{x_T(t) - x_B(t)}{R(t)} \tag{1a}$$

and

$$\cos B(t) = \frac{y_T(t) - y_B(t)}{R(t)} , \tag{1b}$$

where R(t) is the true target range.

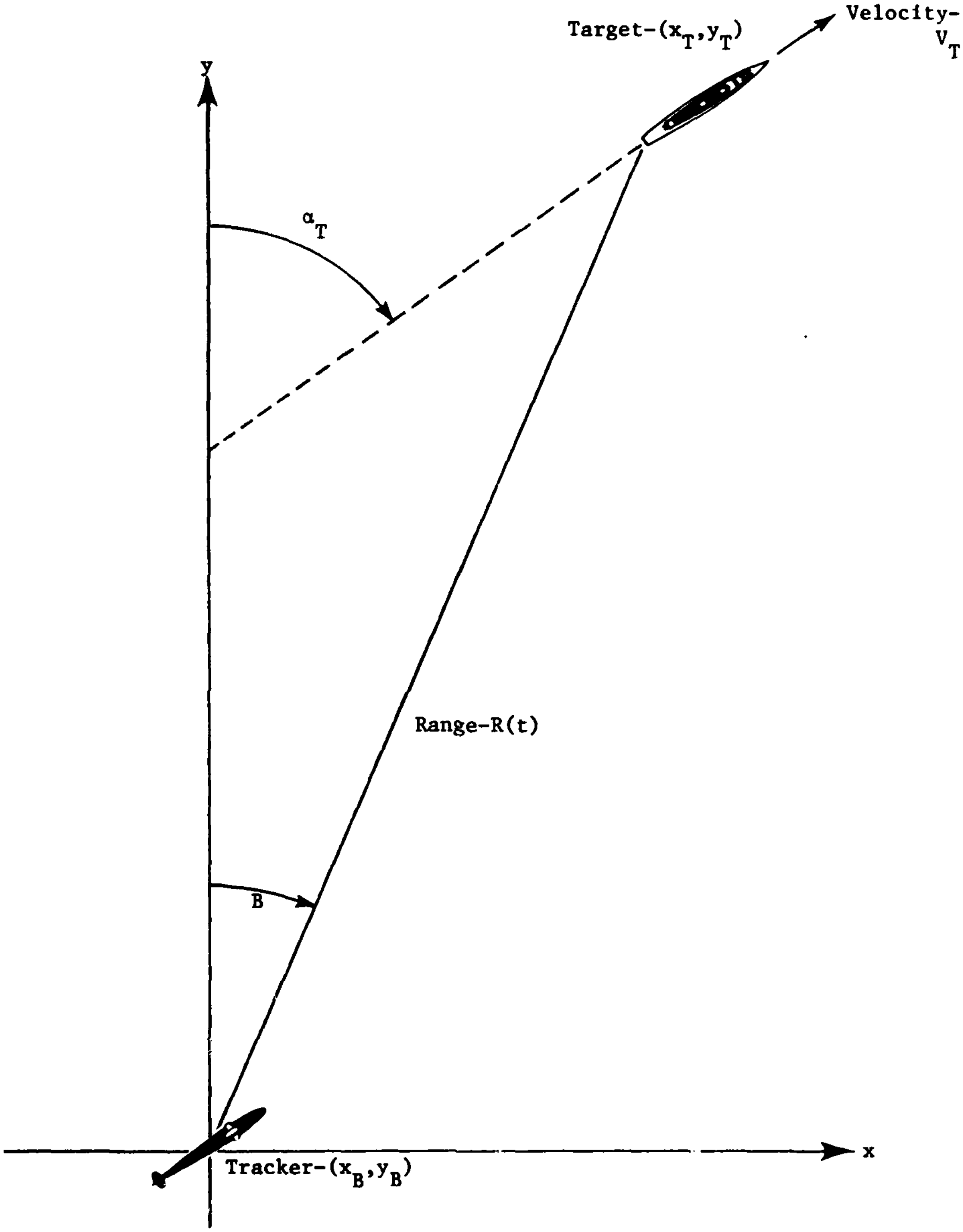

FIGURE 1. PLATFORM AND TARGET GEOMETRY

Assume that the tracker uses a circular hydrophone array to detect acoustic waves radiating from the target. Bearing information is usually obtained by delay-and-sum beamforming. Remember that the direction cosines sin B and cos B are needed to determine the delays used to steer a beam in direction B. Suppose that the beam angle that gives maximum signal energy during the integration time is $\hat{B}$. This is the estimate of the target bearing during the integration time. Let s(t) - sin B(t), c(t) = cos B(t), and $\hat{s}(t)$ and $\hat{c}(t)$ denote the direction cosines corresponding to $\hat{B}$. They can be obtained from the signal processor of the beamformer. Levin[1] and and Hinich and Shaman[2] show that these estimators of sin B and cos B are maximum likelihood estimators if the ambient noise is Gaussian and spatially incoherent. If the array gain is large, moreover, these estimators are approximately Gaussian, unbiased, and independent. For a circular array geometry, the variances of $\hat{s}(t)$ and $\hat{c}(t)$ are equal and are inversely proportional to the energy signal-to-noise ratio (SNR). Expressions for the bearing and direction cosine errors as a function of SNR, aperture, and the number of hydrophones is given by McDonald and Schultheiss,[3] Clay, Hinich, and Shaman,[4] and Hinich.[5]

Suppose that the tracker estimates sin B(t) and cos B(t) at discrete time points $t_n = n\tau$ where τ is the integration time of the beamformer.

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[1]M. J. Levin, "Least-Squares Array Processing for Signals of Unknown Forms," Radio and Electronic Engineer, Vol. 29, pp. 213-222 (1965).

[2]M. J. Hinich and P. Shaman, "Parameter Estimation for an R-Dimensional Plane Wave Observed with Additive Independent Gaussian Errors," The Annals of Mathematical Statistics, Vol. 43, pp. 153-169 (1972).

[3]V. H. MacDonald and P. M. Schultheiss, "Optimum Passive Bearing Estimation," Journal of the Acoustical Society of America, Vol. 46, pp. 37-43 (1969).

[4]C. S. Clay, M. J. Hinich, and P. Shaman, "Error Analysis of Velocity and Direction Measurements of Plane Waves Using Thick Large-Aperture Arrays," Journal of the Acoustical Society of America, Vol. 53, pp. 1161-1166, (1973).

[5]M. J. Hinich, "On Errors in Some Papers on Array Processing," Journal of the Acoustical Society of America, Vol 65, pp. 530-531 (1979).

The statistical expressions derived in the next section are simplified if we set the time origin in the middle of the sampling period; i.e., let n assume integer values $n = -N/2, -N/2 + 1, \ldots, N/2 - 1, N/2$ where N is even. Thus, the sampling interval is $(N + 1)\,\tau$. Assume that τ is selected to ensure that the estimates are uncorrelated over time.

Several important assumptions will now be made about the target motion and the SNR during the sampling period. First, assume that the target's velocity v_T and its heading α_T are constant. Thus, for $-N/2 \leq n \leq N/2$

$$x_T\,(t_n) = x_T\,(0) + v_T\,t_n \sin \alpha_T \tag{2a}$$

and

$$y_T\,(t_n) = y_T\,(0) + v_T\,t_n \cos \alpha_T \quad . \tag{2b}$$

Second, assume that $v_T N\tau << R(0)$ and $v_B N\tau << R(0)$, where v_B is the average speed of the tracker. This implies that the range $R(t)$ is approximately constant during the sampling period. Now let R denote the average range in the sampling period. Finally, assume that the SNR varies sufficiently slowly during this period so that the SNR can reasonably be approximated by a constant. This assumption implies that the variances of the direction cosines are approximately constant. This assumption will be relaxed in a later section.

To simplify notation, select the time unit so that $\tau = 1$ and thus $t_n = n$. It then follows from Equations (1) and (2) and the above assumptions that

$$\hat{s}(n) = R^{-1}\,x_T(0) + (R^{-1}\,v_T \sin \alpha_T)n - R^{-1}\,x_B(n) + \varepsilon_s(n) \tag{3a}$$

and

$$\hat{c}(n) = R^{-1}\, y_T(0) + (R^{-1}\, v_T \cos\, \alpha_T)n - R^{-1}\, y_B(n) + \varepsilon_c(n) \tag{3b}$$

where the errors have the following properties (for circular arrays whose gain is large):

1. $\varepsilon_s(n)$ and $\varepsilon_c(n)$ are independent Gaussian random variables with a common variance denoted σ^2

2. $\varepsilon_s(n)$ and $\varepsilon_s(n')$ (and $\varepsilon_c(n)$ and $\varepsilon_c(n')$) are uncorrelated for all $n \neq n'$

3. The expected values of these errors are approximately zero if $R(t) \approx R$ during the sampling period.

Now, $\hat{s}(n)$, $\hat{c}(n)$, $x_B(n)$ and $y_B(n)$ are observed for $n = -N/2,\ldots,N/2$. Hence, the coordinates $[x_T(n_o),\, y_T(n_o)]$ of the target at time n_o can be estimated.

LEAST SQUARES ESTIMATES OF TRANSFORMED PARAMETERS

The maximum likelihood estimation of these target parameters is facilitated by the following transformations:

$$a_x = R^{-1}\, x_T(0) \qquad a_y = R^{-1}\, y_T(0) \tag{4}$$

$$\beta_x = R^{-1}\, v_T \sin\, \alpha_T \qquad \beta_y = R^{-1}\, v_T \cos\, \alpha_T$$

and

$$b = -R^{-1}\ . \tag{5}$$

Thus, from Equation (3)

$$\hat{s}(n) = a_x + \beta_x n + b\, x_B(n) + \varepsilon_s(n) \tag{6a}$$

and

$$\hat{c}(n) = a_y + \beta_y n + b\, y_B(n) + \varepsilon_c(n) \tag{6b}$$

for $n = -N/2,\ldots,N/2$. Since the errors are Gaussian and have constant variance, the ordinary least-squares (OLS) estimators of a_x, a_y, β_x, and β_y are maximum likelihood.[6] They also have a joint Gaussian distribution. The maximum likelihood estimator of b is a weighted average of $\hat{b}_x$ and $\hat{b}_y$, the OLS estimators of b as computed from Equations (6a) and (6b). The maximum likelihood estimators of the target coordinates are functions of these OLS estimators and are computed from the transforms defined in Equation (4). These estimators are presented in the next section, Target Range and Coordinate Estimates. Before presenting the weights needed to compute $\hat{b}$, the OLS estimators $\hat{a}_x$, $\hat{\beta}_x$, and $\hat{b}_x$ and their statistical properties are presented.

The expressions for the OLS estimators of a_x, β_x, and b_x are simplified if the origin of the coordinate system is placed at the centroid of the platform's track during the sampling period. If this is done, then $\Sigma x_B(n) = \Sigma y_B(n) = 0$. The OLS estimator of a_x is

$$\hat{a}_x = (N + 1)^{-1}\, \Sigma\hat{s}(n) \quad . \tag{7}$$

Its variance is simply

$$\sigma^2_{a_x} = (N + 1)^{-1}\, \sigma^2 \quad . \tag{8}$$

[6] M. G. Kendall, The Advanced Theory of Statistics, Chapter 22, New York: Hafner, Third Edition (1951).

Now define $s(n) = \hat{s}(n) - \bar{s}$, where $\bar{s} = (N + 1)^{-1}\,\Sigma\hat{s}(n)$, the mean of $\hat{s}(n)$. The OLS estimators $\hat{\beta}_x$ and $\hat{b}_x$ can be expressed in vector and matrix form as follows:

$$\begin{pmatrix} \hat{\beta} \\ \\ \hat{b}_x \end{pmatrix} = \begin{pmatrix} \Sigma n^2 & \Sigma n x_B(n) \\ \\ \Sigma n x_B & \Sigma x_B^2(n) \end{pmatrix}^{-1} \begin{pmatrix} \Sigma n s(n) \\ \\ \Sigma x_B(n)\ s(n) \end{pmatrix}$$

The sums are taken from $n = -N/2,\ldots,N/2$. Thus,

$$\hat{\beta}_x = D_x^{-1}\,[\Sigma x_B^2(n)\ \Sigma n s(n) - \Sigma n x_B(n)\ \Sigma x_B(n)\ s(n)] \tag{10}$$

and

$$\hat{b}_x = D_x^{-1}\,[\Sigma n^2\ \Sigma x_B(n)\ s(n) - \Sigma n x_B(n)\ \Sigma n s(n)] \tag{11}$$

where

$$D_x = \Sigma n^2\ \Sigma x_B^2(n) - [\Sigma n x_B(n)]^2 \tag{12}$$

From the triangle inequality, $D_x = 0$ if, and only if, $x_B(n)$ is a linear function of $n = t_n$. Thus, $d_x \neq 0$ if the tracker changes course or speed during the sampling period.

The variance-covariance matrix of $\hat{\beta}_x$ and $\hat{b}_x$ is

$$\frac{\sigma^2}{D_x} \begin{pmatrix} \Sigma x_B^2(n) & -\Sigma n x_B(n) \\ \\ -\Sigma n x_B(n) & \Sigma n^2 \end{pmatrix} . \tag{13}$$

Thus, the variances of $\hat{\beta}_x$ and $\hat{b}_x$ are

$$\sigma^2_{\beta_x} = \sigma^2 D_x^{-1} \Sigma x_B^2(n) \tag{14}$$

and

$$\sigma^2_{b_x} = \sigma^2 D_x^{-1} \Sigma n^2 \quad . \tag{15}$$

It can be shown that $\hat{a}_x$ is uncorrelated with $\hat{\beta}_x$ and $\hat{b}_x$.

It should be obvious from Equations (6a) and (6b) that the OLS estimators $\hat{a}$, $\hat{\beta}$, and $\hat{b}$ are similar to $\hat{a}$, $\hat{\beta}$, and $\hat{b}$ with y (n) and $\hat{c}(n)$ in place place of $x_B(n)$ and $\hat{s}(n)$. Thus $\hat{a}_y = (N + 1)^{-1} \Sigma\hat{c}(n)$.

$$\hat{\beta}_y = D_y^{-1} [\Sigma y_B^2(n) \Sigma nc(n) - \Sigma n y_B(n) \Sigma y_B(n)\ c(n)], \tag{16}$$

and

$$\hat{b}_y = D_y^{-1} [\Sigma n^2 \Sigma y_B(n)c(n) - \Sigma n y_B(n)\Sigma nc(n)], \tag{17}$$

where $c(n) = \hat{c}(n) - \bar{c}$ and

$$D_y = \Sigma n^2 \Sigma y_B^2(n) - [\Sigma n y_B(n)]^2 \quad . \tag{18}$$

Again, $D_y \neq 0$ if the tracker makes a course or speed change.

The estimators $\hat{a}_x$, $\hat{\beta}_x$, and $\hat{b}_x$ are independent of $\hat{a}_y$, $\hat{\beta}_y$, and $\hat{b}_y$ since the errors $[\varepsilon_s(n)]$ are independent of $[\varepsilon_c(n)]$. It then follows from statistical theory[6] that $\hat{b}$, the maximum likelihood estimator of b, is the weighted average

$$(\sigma_{b_x}^{-2} + \sigma_{b_y}^{-2})^{-1}(\sigma_{b_x} \hat{b}_x + \sigma_{b_y}^{-2} \hat{b}_y).$$

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[6]ibid.

Thus

$$\hat{b} = \frac{D_x \hat{b}_x + D_y \hat{b}_y}{D_x + D_y} \tag{19}$$

and its variance is

$$\sigma_b^2 = \sigma^2 (D_x + D_y)^{-1} \Sigma n^2 \quad . \tag{20}$$

From Equations (13) and (19), moreover, the covariance between $\hat{b}$ and $\hat{\beta}_x$ is $-\sigma^2(D_x + D_y)^{-1} \Sigma n x_B(n)$ and the covariance between $\hat{b}$ and $\hat{\beta}_y$ is $-\sigma^2(D_x + D_y)^{-1} \Sigma n y_B(n)$.

TARGET RANGE AND COORDINATE ESTIMATES

Except in some special cases, the variances of the OLS estimators go to zero as $N \rightarrow \infty$. The rate at which this occurs is a function of N and depends upon the form of $[x_B(n), y_B(n)]$; i.e., the form of the platform's track. The variances also go to zero as σ^2, the variance of the bearing direction cosine errors, goes to zero. As is shown in the follow-theorem, the maximum likelihood estimate of the range is biased if $R\sigma_b$ is not small. Assume then that N is sufficiently large and σ is sufficiently small so that $R\sigma_b$ is small.

<u>Theorem 1</u>. $\hat{R} = -1/\hat{b}$ is the maximum likelihood estimator of R. The bias in $\hat{R}$ due to the nonlinear transformation of $\hat{b}$ is

$$E(\hat{R}-R) = R^3\sigma_b^2 + 0\ (R^5\sigma_b^4) \tag{21}$$

and its approximate root mean square error is

$$\text{rmse}\ (\hat{R}) = R^2\sigma_b + 0\ (R^3\sigma_b^2), \tag{22}$$

where σ_b^2 is given by Equation (20). Thus, the bias is an order of magnitude smaller than the rmse when $R\sigma_b << 1$.

Proof. $\hat{b} = b + \varepsilon_b$ is the maximum likelihood estimator of $b = -R^{-1}$. If f(b) is a continuously differentiable function of b, then $f(\hat{b})$ is the maximum likelihood estimator of f(b). Consequently, R is the maximum likelihood estimator of R.

The error ε_b is Gaussian $N(0,\sigma_b^2)$. Thus $E\varepsilon_b = E\varepsilon_b^3 = 0$, $E\varepsilon_b^2 = \sigma_b^2$, and $E\varepsilon_b^4 = 3\sigma_b^4$. Since

$$\frac{\hat{R}}{R} = \frac{1}{1 - R\varepsilon_b} \tag{23}$$

$$= 1 + R\varepsilon_b + R^2\varepsilon_b^2 + R^3\varepsilon_b^3 + \ldots \quad ,$$

it follows that the expected value of $\hat{R}/R$ is

$$E(\hat{R}/R) = 1 + R^2\sigma_b + 0\ [(R\sigma_b)^4], \tag{24}$$

and its mean square error is

$$\text{mse}\ (\hat{R}/R) = R^2\sigma_b^2 + 0\ [(R\sigma_b)^4]. \tag{25}$$

Expressions (21) and (22) follow from Equations (24) and (25).

Given $\hat{R}$, the maximum likelihood estimators of $x_T(n_o)$ and $y_T(n_o)$ can easily be obtained. Accurate estimates of these target coordinates require that $R\sigma_b$ be small.

Theorem 2. The maximum likelihood estimators of $[x_T(n_o),\ y_T(n_o)]$ are

$$\hat{x}_T(n_o) = \hat{R}(\hat{A}_x + n_o\hat{\beta}_x)$$

and (26)

$$\hat{y}_T(n_o) = \hat{R}(a_y + n_o\beta_y).$$

They are independent. For large N, their distributions are approximately Gaussian with zero means and root mean square errors

$$\text{rmse }(\hat{x}_T) \approx R\sigma[\frac{1}{N} + x_T^2(n_o)\frac{\Sigma n^2}{D_x + D_y} - 2n_o x_T(n_o)\frac{\Sigma n x_B(n)}{D_x + D_y} + n_o^2 \frac{\Sigma x_B^2(n)}{D_x}]^{1/2}$$

and

$$\text{rmse }(\hat{y}_T) \approx R\sigma[\frac{1}{N} + y_T^2(n_o)\frac{\Sigma n^2}{D_x + D_y} - 2n_o y_T(n_o)\frac{\Sigma n y_B(n)}{D_x + D_y} + n_o^2 \frac{\Sigma y_B^2(n)}{D_y}]^{1/2}.$$

Proof: Write $\hat{a}_x = a_x + \varepsilon_{a_x}$, $\hat{\beta}_x = \beta_x + \varepsilon_{\beta_x}$, and $\hat{b} = b + \varepsilon_b$. ε_{a_x} and ε_{β_x} are uncorrelated Gaussian errors. The error ε_b is correlated with ε_{β_x}. Apply Equations (4) and (23) to Equation (26) to obtain the following approximation (in the errors):

$$\hat{x}_T(n_o) \approx R(1 + R\varepsilon_b)R^{-1}x_T(0) + \varepsilon_{a_x} + n_o(R^{-1}v_T\sin\alpha_T + \varepsilon_{\beta_x})$$

$$= (1 + R\varepsilon_b)[x_T(n_o) + R\varepsilon_{a_x} + Rn_o\varepsilon_{\beta_x}]$$

$$\approx x_T(n_o) + R(\varepsilon_{a_x} + x_T(n_o)\varepsilon_b + n_o\varepsilon_{\beta_x}) \quad . \tag{27}$$

Apply Equations (14), (20), and the expression for the covariance between $\hat{b}$ and $\hat{\beta}_x$ to Equation (27) to derive the expression for rmse $(\hat{x}_T)$. Derive the expression for rmse $(\hat{y}_T)$ in a similar manner.

NON-CONSTANT BEARING VARIANCE

Now relax the restriction that the SNR is constant during the sampling period. There are several methods for estimating the SNR associated with a specific bearing estimate $\hat{B}$. One method uses output from the beamformer. If the noise field is isotropic in the sector $\hat{B} - \delta < B < \hat{B} + \delta$, then the average of the energy in the beams spanning this 2δ wide sector (excluding $\hat{B}$) is an estimate of the noise field energy. Let $e(\delta)$ denote this estimate. The energy in the $\hat{B}$ beam is an estimate of signal plus noise. Let $e(\hat{B})$ denote this estimate. Then

$$\hat{\rho} = \frac{e(\hat{B})}{e(\delta)} - 1 \quad . \tag{28}$$

A somewhat more precise estimate uses the average of the coherence between pairs of sensor channels. To illustrate this method, let $\hat{\gamma}^2$ denote the average estimated square coherence between two hydrophone channels over the bandwidth of the signal. Then the SNR is estimated by

$$\frac{\hat{\gamma}^2}{1-\hat{\gamma}^2} \quad . \tag{29}$$

The maximum likelihood estimators are functions of the variance of the direction cosines if these variances change during the sampling period. Since the variance depends on the SNR, it must be estimated every time a bearing is obtained if one desires to approximate the statistical properties of maximum likelihood estimators.

The adjustment for a changing error variance is easy using the linear model approach; i.e., use weighted least squares.[7] Simply divide the independent variables $[n, x_B(n), \text{and } y_B(n)]$ and the dependent variables $[\hat{s}(n)$ and $\hat{c}(n)]$ by the estimated variance for each n. Since the variance is inversely proportional to the SNR given a narrowband signal, the adjustment is made by multiplying the variables by $\hat{\rho}(n)$, a consistent estimator of the SNR at time $t = n\tau$. All the expressions for the estimators and their properties given in the previous sections hold if n, $x_B(n)$, $y_B(n)$, $\hat{s}(n)$, and $\hat{c}(n)$ are multiplied by $\hat{\rho}(n)$. If the true SNR for each n were known, then the adjusted estimators would be maximum likelihood.

It is important to remember that $\hat{\rho}$ is an estimate of the SNR and not the true SNR for any type of estimator. The error inherent in $\hat{\rho}$ increases the error in the parameter estimates. Consequently, it is generally better to use the ordinary least squares approach rather than weighted least squares (the multiplication adjutment) when the SNR is slowly varying during the sampling period.

The following ad hoc compromise between ordinary least squares and weighted least squares estimation may provide a more robust estimation method. The compromise rejects a bearing estimate if its estimated SNR is below some threshold value. Suppose, for example, that the SNR for $\hat{s}(2)$ and $\hat{c}(2)$ is below the threshold. Then delete $\hat{s}(2)$, $\hat{c}(2)$, $x_B(2)$, $y_B(2)$, and 2 from the sums in the expressions for $\hat{\beta}_x$, $\hat{\beta}_y$, $\hat{b}_x$, and $\hat{b}_y$. The target parameters are estimated from those bearings whose SNR exceed the threshold, using the ordinary least squares approach.

SIMULATION

It is assumed the target is moving at a constant velocity of 90 yards/ 15 seconds or 0.36 kyd/minute on a fixed heading of 90 degrees. For

[7] F. A. Graybill, An Introduction to Linear Statistical Models, Vol. 1, New York: John Wiley & Sons, Inc. (1961).

convenience, the tracker's speed is set at the same rate. This is not necessary for the results to hold; it merely facilitates the calculations.

Upon observing the target, the tracker begins its tracking maneuver. In the simulation, the track is circular (Figure 2). The radius of the circle is such that the tracker completes two circles during the tracking sequence at the constant velocity of 0.36 kyd/minute. This particular track was chosen for several reasons. First, it is symmetric; that is, one circle is completed in each half of the tracking sequence. Thus, the centroid of the tracking sequence is the origin of the coordinate system. Second, the track of two circles yields two estimates of the target's bearing and, hence, the estimated bearing sine and cosine, at each point where the tracker takes a reading. This enables the OLS routine to estimate more accurately the target's heading since any difference in the two estimates of the target's bearing is due to a change in the target's position. And finally, a circular track minimizes the expected variance of the target's range for a smooth track when the tracker has no prior knowledge of the target's heading. In this simulation, 45 observations of the target's estimated bearing are taken. This number is strictly arbitrary although the routine requires that the number of observations be an odd integer. Hence, the radius of the circular track is $45/4\pi$ or 0.322 yard. The tracker, therefore, travels 3.96 kyds after having first observed the target taking an observation every 15 seconds or every 90 yards.

Each estimate of the target's bearing is used to calculate an estimate of $\hat{s}(n)$ and $\hat{c}(n)$. The resulting 45 estimates are used to estimate the parameters of Equations (6a) and (6b) by means of OLS. The estimated parameters obtained are used to calculate estimates of the target's heading, range, and velocity. The estimated heading is $\hat{\alpha}_T = \pi/2 - \tan^{-1}(\hat{\beta}_y/\hat{\beta}_x)$. It is computed in this manner because $\hat{\beta}_y$ is close to zero, and thus small variations in $\hat{\beta}_y$ produce large fluctuations in the estimate $\tan^{-1}(\hat{\beta}_x/\hat{\beta}_y)$. The estimated range, $\hat{R}$, is calculated according to Equation (5). However, early work with the simulation suggested that use of weighted estimator $\hat{b}$ provided highly variable and unrealistic estimates of the range. This is due to the fact that $\hat{b}_y$, like $\hat{\beta}_y$, is close to zero and is often estimated

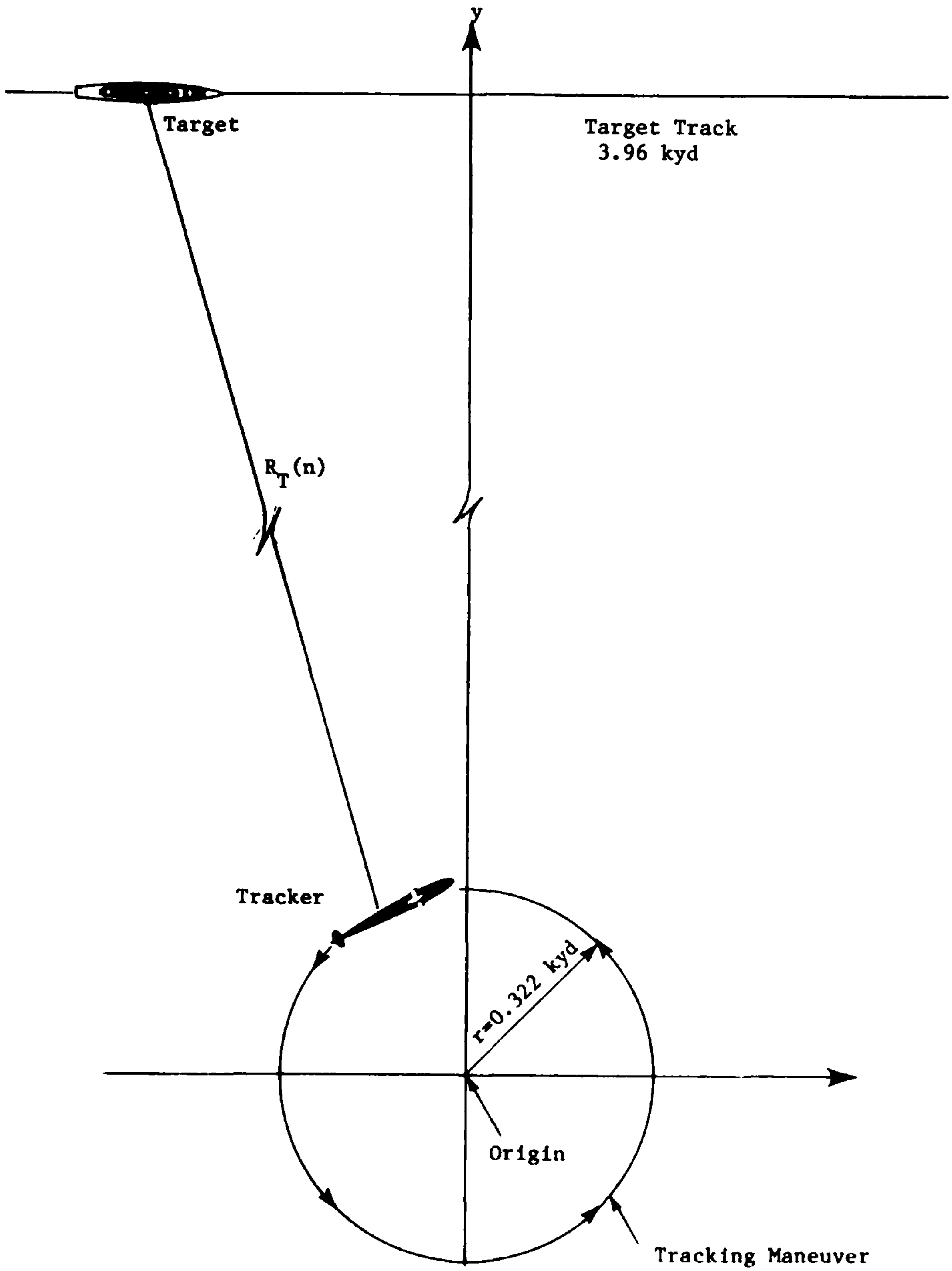

FIGURE 2. SIMULATION GEOMETRY

as a positive fraction. The smaller of $\hat{b}_x$ and $\hat{b}_y$ is selected to estimate the range. The program consistently selected $\hat{b}_x$ since α_T is 90 degrees. The weighted estimator $\hat{b}$ is only optimal when the sample size is large. The sample size is not large in this simulation. The results in Table 1 are derived by using the weighted estimator of range, $-\hat{b}^{-1}$. Table 2 presents the results obtained by using the ad hoc estimator, $-\hat{b}_x^{-1}$. The estimated velocity $\hat{v}_T$, is estimated by $\hat{v}_T = [\hat{R}^2(\hat{\beta}_x^2 + \hat{\beta}_y^2)]^{1/2}$.

These estimates are then used to compute the estimated target coordinates at the end of the tracking sequence $\hat{x}_T(N/2)$ and $\hat{y}_T(N/2)$ according to the formula of Theorem 2. In addition, the theoretical rmse's of these values are calculated. They are denoted $rmse_T(\hat{x}_T)$ and $rmse_T(\hat{y}_T)$ in the tables and are presented for comparison to the actual values found in the simulation.

The simulation is repeated 400 times. The tables present the actual values as well as the mean, mean bias, standard deviation, and the root mean square error (rmse) for each of the estimated variables. The estimated variables are (1) the target's estimated coordinates at the end of the tracking sequence, $\hat{x}_T(N/2)$ and $\hat{y}_T(N/2$, (2) the target's estimated average range during tracking, $\hat{R}$, (3) the target's estimated heading, $\hat{\alpha}_T$; and (4) the target's estimated velocity, $\hat{v}_T$. In addition, the actual maximum and minimum range to the target during tracking is presented.

Compare the estimates in Table 2, which are obtained using the ad hoc estimator of range, $-\hat{b}_x^{-1}$, with those of Table 1, which were obtained using the weighted estimator of range. It is apparent that the use of the weighted estimator significantly increases both the bias and variance of all the estimates obtained. The exception is the estimated heading $\hat{\alpha}_T$. This result follows from the fact that the weighted estimator consistently over estimates the range which is used to calculate all estimates except that of the estimated heading $\hat{\alpha}_T$.

TABLE 1

RESULTS OBTAINED USING WEIGHTED ESTIMATOR OF RANGE $\sigma_\varepsilon = 0.20$

End of Track	Actual	Mean	Mean Bias	Standard Deviation	RMSE
$\hat{x}_T(N/2)$	1.9799986	3.6655350	1.6855354	0.2367992	1.7020874
$\hat{y}_T(N/2)$	19.7999878	36.7997894	16.9997864	0.1459860	17.0004120
$R_T(N/2)$	19.8363953	36.8514709	17.0150604	2.3269196	17.1734467
$\hat{\alpha}_T$	90.0000000	89.7862091	-0.2137576	0.5106176	0.5535945
V_T	0.3600000	0.6682853	0.3082853	0.0427839	0.3112399

Maximum and minimum range during tracking: 20.181183 and 19.485062

$N = 45$ $\bar{R} = 19.80$ $rmse_T(\hat{x}_T) = 0.069$ $rmse_T(\hat{y}_T) = 0.667$

TABLE 2

RESULTS OBTAINED USING AD HOC ESTIMATOR OF RANGE $\sigma_\varepsilon = 0.20$

End of Track	Actual	Mean	Mean Bias	Standard Deviation	RMSE
$\hat{x}_T(N/2)$	1.9799986	1.9686899	-0.0113090	0.0916492	0.0923442
$\hat{y}_T(N/2)$	19.7999878	19.7851410	-0.0148508	0.0908208	0.0920270
$R_T(N/2)$	19.8363953	19.8159637	-0.0204318	0.8945770	0.8948103
$\hat{\alpha}_T$	90.0000000	89.8274689	-0.1725032	0.5489491	0.5754150
V_T	0.3600000	0.3588170	-0.0011830	0.0168327	0.0168743

Maximum and minimum range during tracking: 20.181183 and 19.485062

$N = 45$ $\bar{R} = 19.80$ $rmse_T(\hat{x}_T) = 0.069$ $rmse_T(\hat{y}_T) = 0.667$

Table 3 shows the results when the bearing sine and cosine errors are doubled from 0.2 degree in Table 2 to 0.4 degree. The doubling of the errors approximately doubles the rmse's of all the estimated variables. The rmse approximates the asymptotic bias of the estimator.

Tables 4 and 5 present the simulation results when the range to target is doubled but the bearing errors are 0.2 and 0.4 degree. With the exception of the estimated range, the rmse's of the estimates are approximately equal to those obtained when the errors were doubled. Increasing either the range to target or the errors on the bearing estimates by a multiplicative constant, k, increases the bias by approximately k. This empirical relationship does not hold for the estimated range. Although the rmse of the estimated range does approximately double when the bearing errors are doubled, it increases by approximately a factor of 4 when the range to target is doubled.

The results of the simulation support the theory presented in the paper. These results are a function of the particular tracking sequence chosen as well as the target's track. The circular track as well as the 90-degree heading were chosen for the simulations because of their simplicity.

NOTE: THESE SYMBOLS ARE USED IN THE TABLES AND ARE IDENTIFIED AGAIN FOR CLARITY.

rmse	root mean square error of.......
(x_T, y_T)	coordinates of the target
(x_B, y_B)	coordinates of the tracker
N	sample size
$\bar{R}$	average range to target
σ_ε	rmse of ε
ε	error term of regression
R_T	range to target
α_T	heading to target
V_T	velocity of target
	statistical estimator of a parameter

TABLE 3

RESULTS OBTAINED USING AD HOC ESTIMATOR OF RANGE $\sigma_{\varepsilon} = 0.40$

End of Track	Actual	Mean	Mean Bias	Standard Deviation	RMSE
$\hat{x}_T(N/2)$	1.9799986	1.9826841	0.0026848	0.2073205	0.2073379
$\hat{y}_T(N/2)$	19.7999878	19.9087372	0.1087440	0.1365973	0.1745969
$R_T(N/2)$	19.8363953	19.9413757	0.1049771	2.0171242	2.0198536
$\hat{\alpha}_T$	90.0000000	89.8330841	-0.1668751	1.0594177	1.0724792
V_T	0.3600000	0.3613726	0.0013726	0.0376254	0.0376504

Maximum and minimum range during tracking: 20.181183 and 19.485062

N = 45 $\bar{R} = 19.80$ $rmse_T(\hat{x}_T) = 0.138$ $rmse_T(\hat{y}_T) = 1.334$

TABLE 4

RESULTS OBTAINED USING AD HOC ESTIMATOR OF
RANGE $\sigma_{\varepsilon} = 0.20$ AND $\bar{R} = 39.60$

End of Track	Actual	Mean	Mean Bias	Standard Deviation	RMSE
$\hat{x}_T(N/2)$	1.9799986	1.9804792	0.0004807	0.2079040	0.2079045
$\hat{y}_T(N/2)$	39.5999756	39.7122803	0.1123022	0.1367894	0.1769834
$R_T(N/2)$	39.6180878	39.7253571	0.1072704	3.9831715	3.9846144
$\hat{\alpha}_T$	90.0000000	89.8725433	-0.1274239	1.1198845	1.1271105
V_T	0.3600000	0.3608412	0.0008411	0.0373895	0.0373989

Maximum and minimum range during tracking: 39.951111 and 39.281448

N = 45 $rmse_T(\hat{x}_T) = 0.138$ $rmse_T(\hat{y}_T) = 2.653$

TABLE 5

RESULTS OBTAINED USING AD HOC ESTIMATOR OF RANGE $\sigma_{\varepsilon} = 0.40$ AND $\bar{R} = 39.60$

End of Track	Actual	Mean	Mean Bias	Standard Deviation	RMSE
$\hat{x}_T(N/2)$	1.9799986	2.0484772	0.0684777	0.4705369	0.4754934
$\hat{y}_T(N/2)$	39.5999756	41.0848694	1.4848890	0.2057871	1.4990807
$R_T(N/2)$	39.6180878	41.1012878	1.4831972	9.1944504	9.3133106
$\hat{\alpha}_T$	90.0000000	89.9536896	-0.0462814	2.1883059	2.1887951
V_T	0.3600000	0.3733910	0.0133910	0.0855601	0.0866016

Maximum and minimum range during tracking: 39.951111 and 39.281448

$N = 45$ $\quad$ $\mathrm{rmse}_T(\hat{x}_T) = 0.276$ $\quad$ $\mathrm{rmse}_T(\hat{y}_T) = 5.306$

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